Lexical Analysis (Tokenizing)

COMP 3002

School of Computer Science

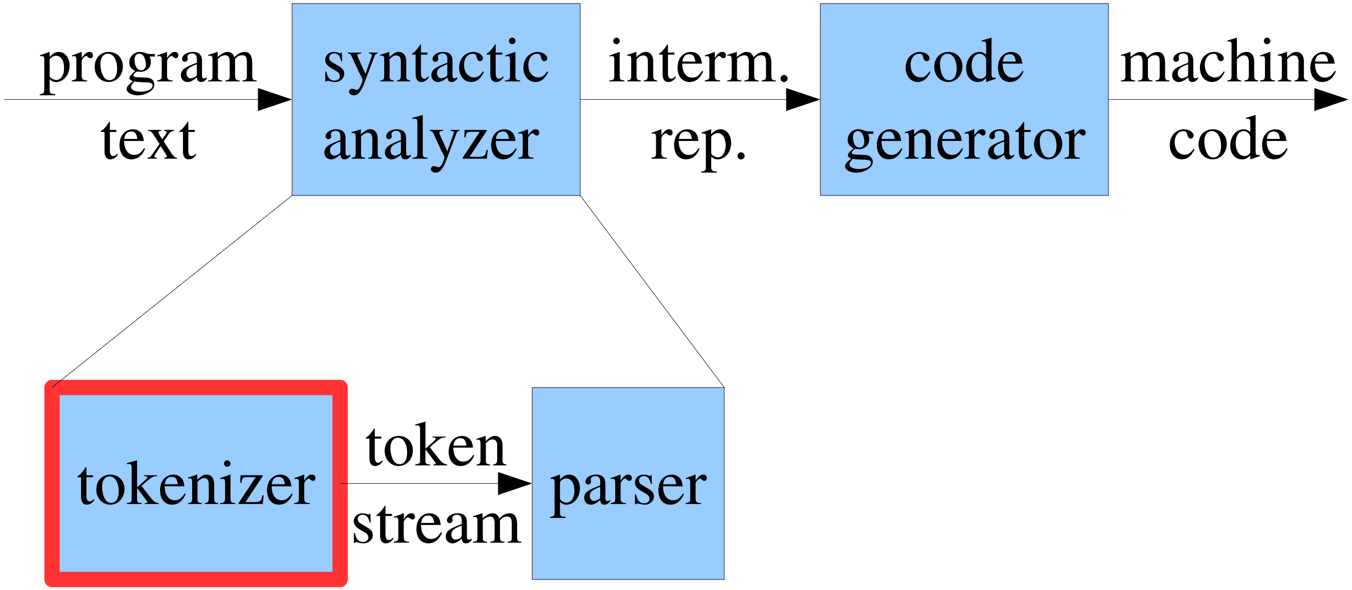
Carleton
UNIVERSITY
Canada's Capital University

List of Acronyms

- RE - regular expression
- FSM - finite state machine
- NFA - non-deterministic finite automata
- DFA - deterministic finite automata

Carleton UNIVERSITY
Canada's Capital University

The Structure of a Compiler



Purpose of Lexical Analysis

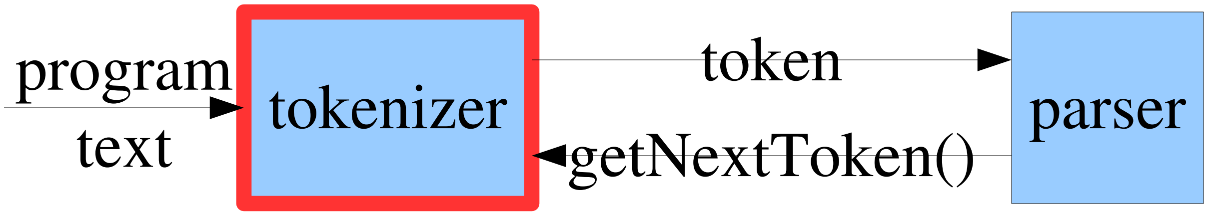
- Converts a character stream into a token stream

```
int main(void) {
  for (int i = 0;
        i < 10;
        i++) { ...
```

Carleton UNIVERSITY
Canada's Capital University

How the Tokenizer is Used

- Usually the tokenizer is used by the parser, which calls the `getNextToken()` function when it wants another token
- Often the tokenizer also includes a `pushBack()` function for putting the token back (so it can be read again)



Other Tokenizing Jobs

- Input reading and buffering
- Macro expansion (C's #define)
- File inclusion (C's #include)
- Stripping out comments

Carleton UNIVERSITY
Canada's Capital University

Tokens, Patterns, and Lexemes

- A *token* is a pair
  - token name (e.g., VARIABLE)
  - token value (e.g., "myCounter")
- A *lexeme* is a sequence of program characters that form a token
  - (e.g., "myCounter")
- A *pattern* is a description of the form that the lexemes of a token may take
  - e.g., character strings including A-Z, a-z, 0-9, and _

Carleton
UNIVERSITY
Canada's Capital University

A History Lesson

- Usually tokens are easy to recognize even without any context, but not always
- A tough example from Fortran 90:

```
DO 5 I = 1.25
<variable, "DO5I"> <assign> <number,"1.25">
```

```
DO 5 I = 1,25
<do> <number, "5"> <variable, "I">
<assign> <number, "1"> <comma> <number, "25">
```

Carleton UNIVERSITY
Canada's Capital University

Lexical Errors

- Sometimes the current prefix of the input stream does not match any pattern
  - This is an error and should be logged
- The lexical analyzer may try to continue by
  - deleting characters until the input matches a pattern
  - deleting the first input character
  - adding an input character
  - replacing the first input character
  - transposing the first two input characters

Carleton UNIVERSITY
Canada's Capital University

Exercise

- Circle the lexemes in the following programs

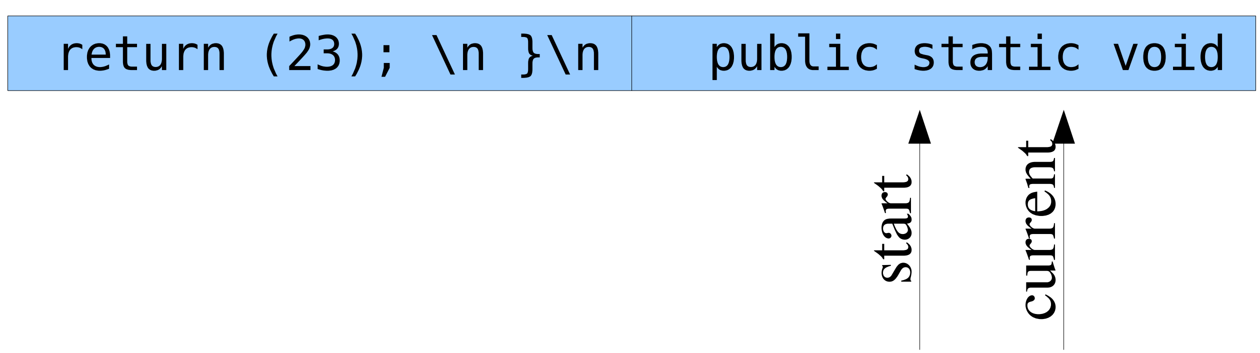
```
public static void main(String args[]) {
  System.println("Hello World!");
}
```

```
float max(float a, float b) {
  return a > b ? a : b;
}
```

Carleton UNIVERSITY
Canada's Capital University

Input Buffering

- Lexemes can be long and the pushBack function requires a mechanism for pushing them back
- One possible mechanism (suggested in the textbook) is a double buffer
- When we run off the end of one buffer we load the next buffer



Tokenizing (so far)

- What a tokenizer does
  - reads character input and turns it into tokens
- What a token is
  - a token name and a value (usually the lexeme)
- How to read input
  - use a double buffer if some lookahead is necessary
- How does the tokenizer recognize tokens?
- How do we specify patterns?

Carleton UNIVERSITY
Canada's Capital University

Where to Next?

- We need a formal mechanism for defining the patterns that define tokens
- This mechanism is formal language theory
- Using formal language theory we can make tokenizers without writing any actual code

Carleton UNIVERSITY
Canada's Capital University

Strings and Languages

- An *alphabet* Σ is a set of symbols
- A *string* S over an alphabet Σ is a finite sequence of symbols in Σ
- The *empty string*, denoted ε, is a string of length 0
- A *language* L over Σ is a countable set of strings over Σ

Carleton UNIVERSITY
Canada's Capital University

Examples of Languages

- The empty language $L = \emptyset$
- The language $L = \{\varepsilon\}$ containing only the empty string
- The set L of all syntactically correct C programs
- The set L of all valid variable names in Java
- The set L of all grammatically correct english sentences

Carleton UNIVERSITY
Canada's Capital University

String Concatenation

- If x and y are strings then the *concatenation* of x and y, denoted xy, is the string formed by appending y to x
- Example
  - x = "dog"
  - y = "house"
  - xy = "doghouse"
- If we treat concatenation as a "product" then we get *exponentiation*:
  - x^2 = "dogdog"
  - x^3 = "dogdogdog"

Carleton UNIVERSITY
Canada's Capital University

Operations on Languages

- We can form complex languages from simple ones using various operations
- Union: $L \cup M$ (also denoted $L \mid M$)
  - $L \cup M = \{ s : s \in L \text{ or } s \in M \}$
- Concatenation
  - $LM = \{ st : s \in L \text{ and } t \in M \}$
- Kleene Closure L^*
  - $L^* = \{ L^i : i = 0, 1, 2, \ldots \}$
- Positive Closure L^+
  - $L^* = \{ L^i : i = 1, 2, 3, \ldots \}$

Carleton UNIVERSITY
Canada's Capital University

Carleton UNIVERSITY
Canada's Capital University

Some Example

- L = { A,B,C,...Z,a,b,c,...z }
- D = { 0,1,2,3,4,5,6,7,8,9 }
- $L \cup D$
- LD
- L^4
- L^*
- $L(L \cup D)^*$
- $D+$

Regular Expressions

- Regular expressions provide a notation for defining languages
- A regular expression *r* denotes a language $L(r)$ over a finite alphabet Σ
- Basics:
  - ε is a RE and $L(\varepsilon) = \{ \varepsilon \}$
  - For each symbol *a* in Σ, *a* is a RE and $L(a) = \{ a \}$

Carleton UNIVERSITY
Canada's Capital University

Regular Expression Operators

- Suppose r and s are regular expressions
- Union (choice)
  - (r)|(s) denotes $L(r) \cup L(s)$
- Concatenation
  - (r)(s) denotes $L(r)\ L(s)$
- Kleene Closure
  - r* denotes $(L(r))^*$
- Parenthesization
  - (r) denote $L(r)$
  - Used to enforce specific order of operations

Carleton UNIVERSITY
Canada's Capital University

Order of Operations in REs

- To avoid too many parentheses, we adopt the following conventions
  - The * operator has the highest level of precedence and is left associative
  - Concatenation has second highest precedence and is left associative
  - The | operator has lowest precedence and is left associative

Carleton UNIVERSITY
Canada's Capital University

Binary Examples

- For the alphabet $\Sigma = \{ a,b \}$
  - a|b denotes the language { a, b }
  - (a|b)(a|b) denotes the langage { aa, ab, ba, bb }
  - a* denotes { ε, a, aa, aaa, aaaa, ... }
  - (a|b)* denotes all possible strings over Σ
  - a|a*b denotes the language { a, b, ab, aab, aaab, ... }

Carleton UNIVERSITY
Canada's Capital University

Regular Definitions

- REs can quickly become complicated
- *Regular definitions* are multiline regular expressions
- Each line can refer to any of the preceding lines *but not to itself or to subsequent lines*

Carleton UNIVERSITY

Canada's Capital University

```
letter_ = A|B|...|Z|a|b|...|z|_
digit   = 0|1|2|3|4|5|6|7|8|9
id      = letter_(letter_|digit)*
```

Regular Definition Example

- Floating point number example
  - Accepts 42, 42.314159, 42.314159E+23, 42E+23, 42E23, …

```
digit   = 0|1|2|3|4|5|6|7|8|9
digits  = digit digit*
optionalFraction = . digits | ε
optionalExponent = (E (+|-|ε) digits) | ε
number = digits optionalFraction optionalExponent
```

Carleton UNIVERSITY
Canada's Capital University

Exercises

- Write regular definitions for
  - All strings of lowercase letters that contain the five vowels in order
  - All strings of lowercase letters in which the letters are in ascending lexicographic order
  - Comments, consisting of a string surrounded by /* and */ without any intervening */

Carleton UNIVERSITY
Canada's Capital University

Extension of Regular Expressions

- There are also several time-saving extensions of REs
- One or more instances
  - `r+ = rr*`
- Zero or one instance
  - `r? = r|ε`
- Character classes
  - `[abcdef] = (a|b|c|d|e|f)`
  - `[A-Za-z] = (A|B|C|...|Y|Z|a|b|c|...|y|z)`
- Others
  - See page 127 of the text for more common RE shorthands

Some Examples

```
digit   = [0-9]
digits  = digit+
number = digits (. digits)? (E[+-]? digits)?
```

```
letter_ = [A-Za-z_]
digit   = [0-9]
variable= letter_ (letter|digit)*
```

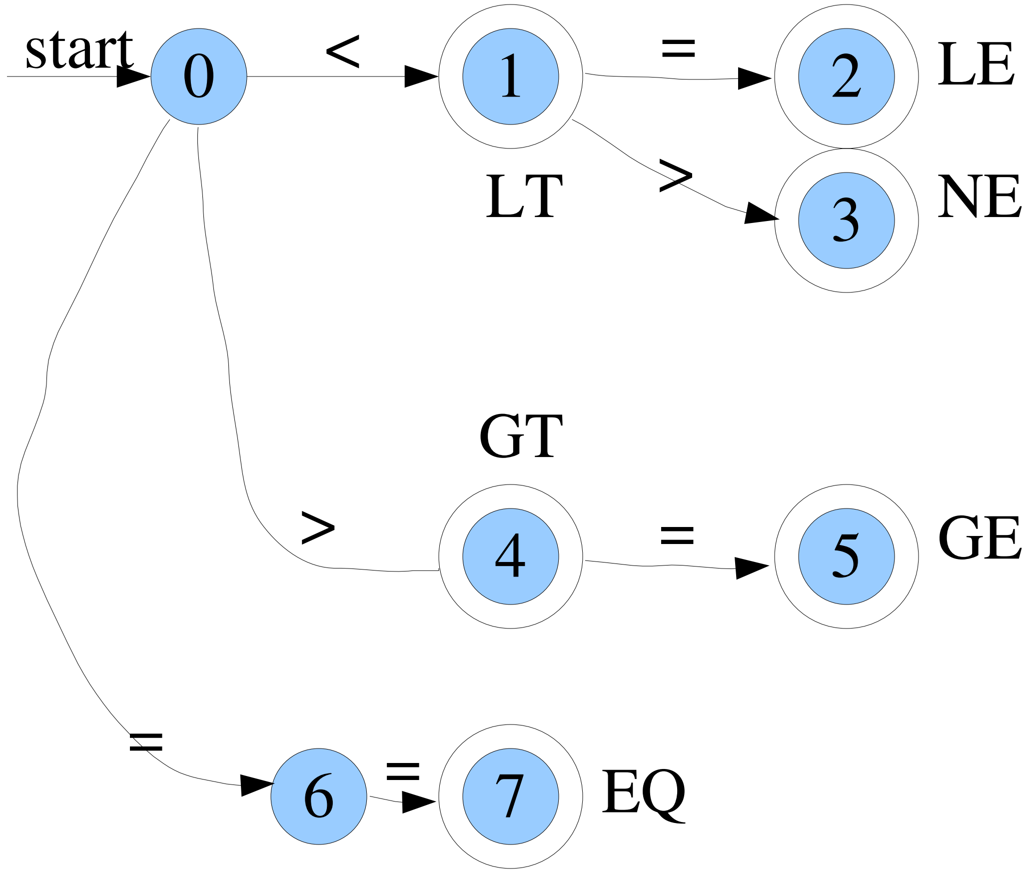
Carleton UNIVERSITY
Canada's Capital University

Recognizing Tokens

- We now have a notation for patterns that define tokens
- We want to make these into a tokenizer
- For this, we use the formalism of finite state machines

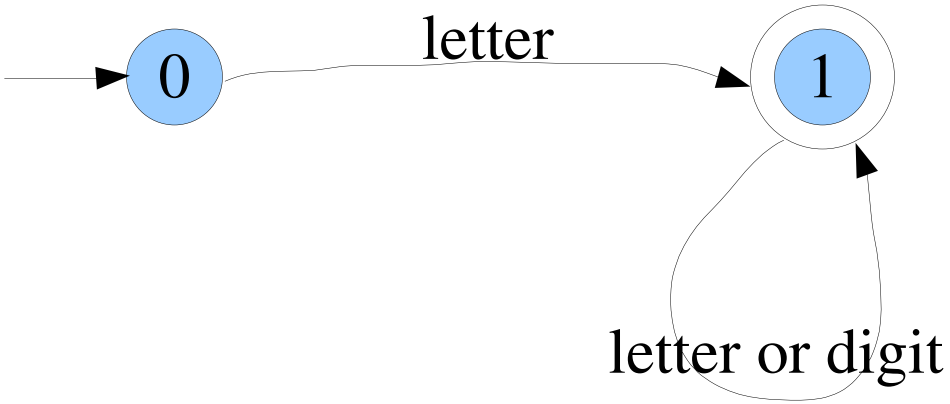
An FSM for Relational Operators

- relational operators <, >, <=, >=, ==, <>



FSM for Variable Names

```
letter_ = [A-Za-z_]
digit   = [0-9]
variable= letter_ (letter|digit)*
```



Carleton UNIVERSITY
Canada's Capital University

FSM for Numbers

- Build the FSM for the following:

```
digit   = [0-9]
digits  = digit+
number = digits (. digits)? ((E|e) digits)?
```

Carleton UNIVERSITY
Canada's Capital University

NumReader.java

- Look at NumReader.java example
  – Implements a token recognizer using a switch statement

Carleton UNIVERSITY
Canada's Capital University

The Story So Far

- We can write tokens types as regular expressions
- We want to convert these REs into (deterministic) finite automata (DFAs)
- From the DFA we can generate code
  - A single while loop containing a large switch statement
    - Each state in S becomes a case
  - A table mapping $S \times \Sigma \rightarrow S$
    - (current state,next symbol)→(new state)
  - A hash table mapping $S \times \Sigma \rightarrow S$
  - Elements of Σ may be grouped into character classes

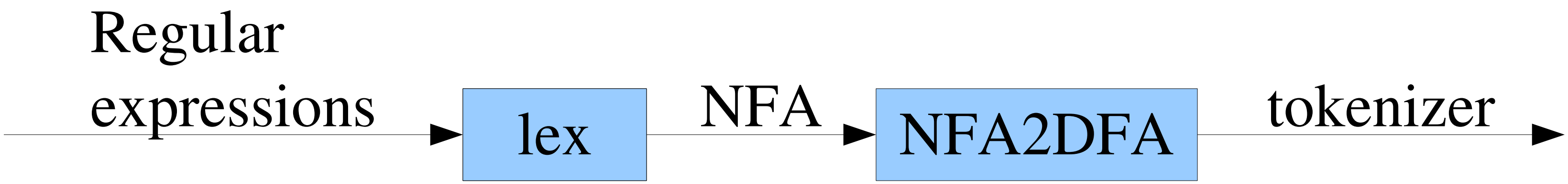
Carleton UNIVERSITY
Canada's Capital University

NumReader2.java

- Look at NumReader2.java example
  - Implements a tokenizer using a hashtable

Automatic Tokenizer Generators

- Generating FSMs by hand from regular expressions is tedious and error-prone
- Ditto for generating code from FSMs
- Luckily, it can be done automatically



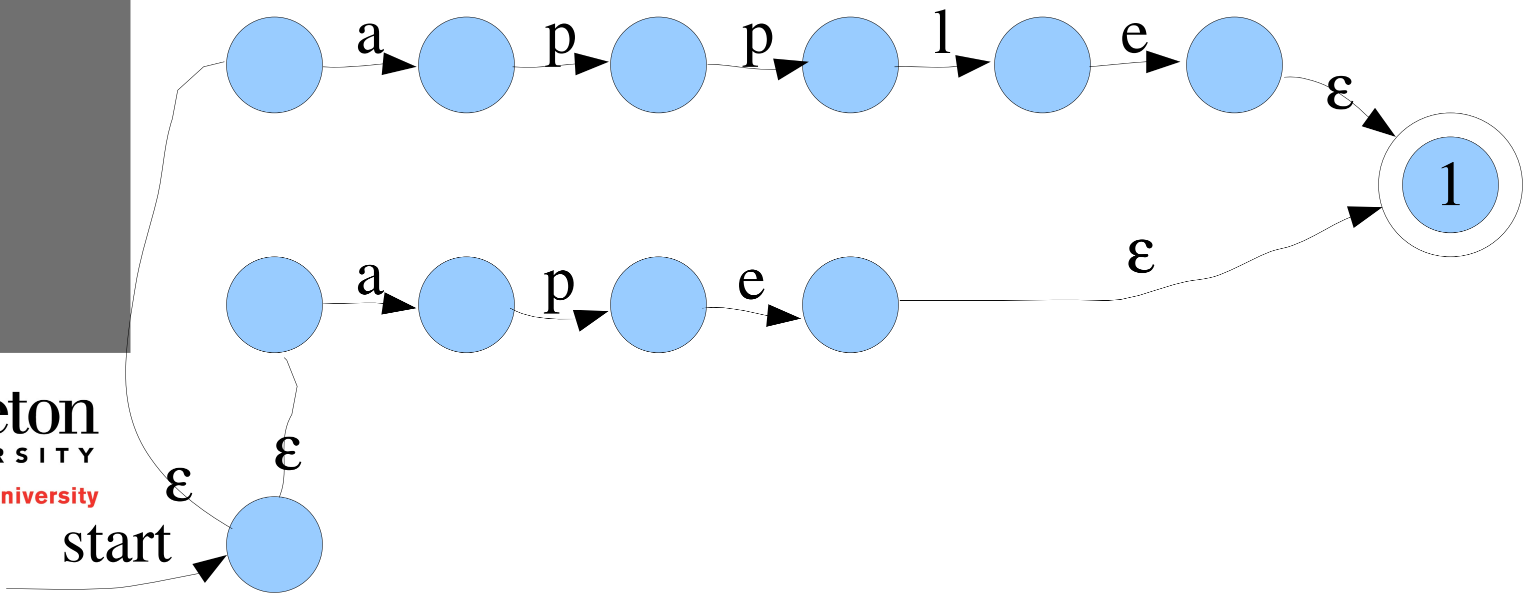
Non-Deterministic Finite Automata

- An NFA is a finite state machine whose edges are labelled with subsets of Σ
- Some edges may be labelled with ε
- The same labels may appear on two or more outgoing edges at a vertex
- An NFA accepts a string *s* if *s* defines any path to any of its accepting states

Carleton UNIVERSITY
Canada's Capital University

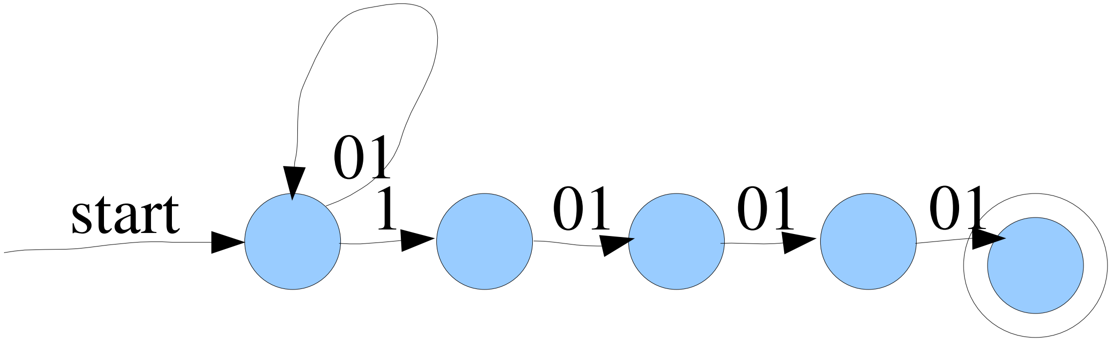
NFA Example

- NFA that accepts apple or ape



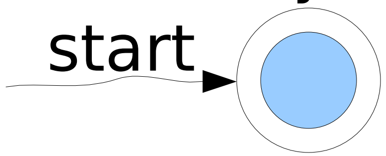
NFA Example

- NFA that accepts any binary string whose 4 last value is 1

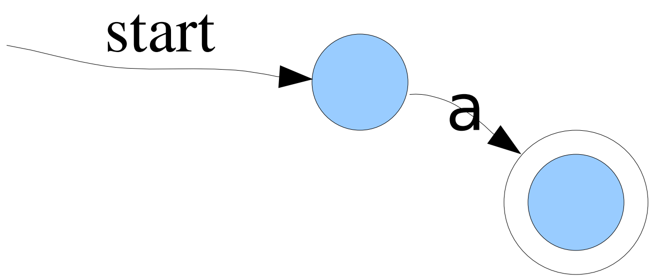


From Regular Expression to NFA

- Going from a RE to a NFA with one accepting state is easy
- ε

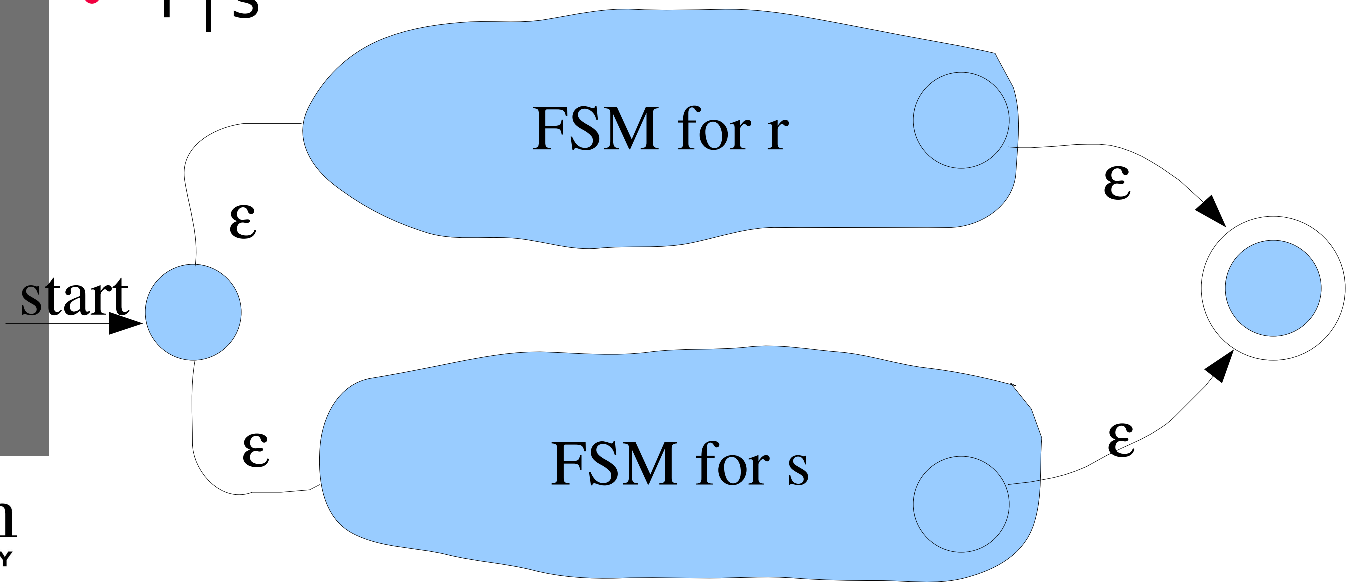


- a



Union

- r|s



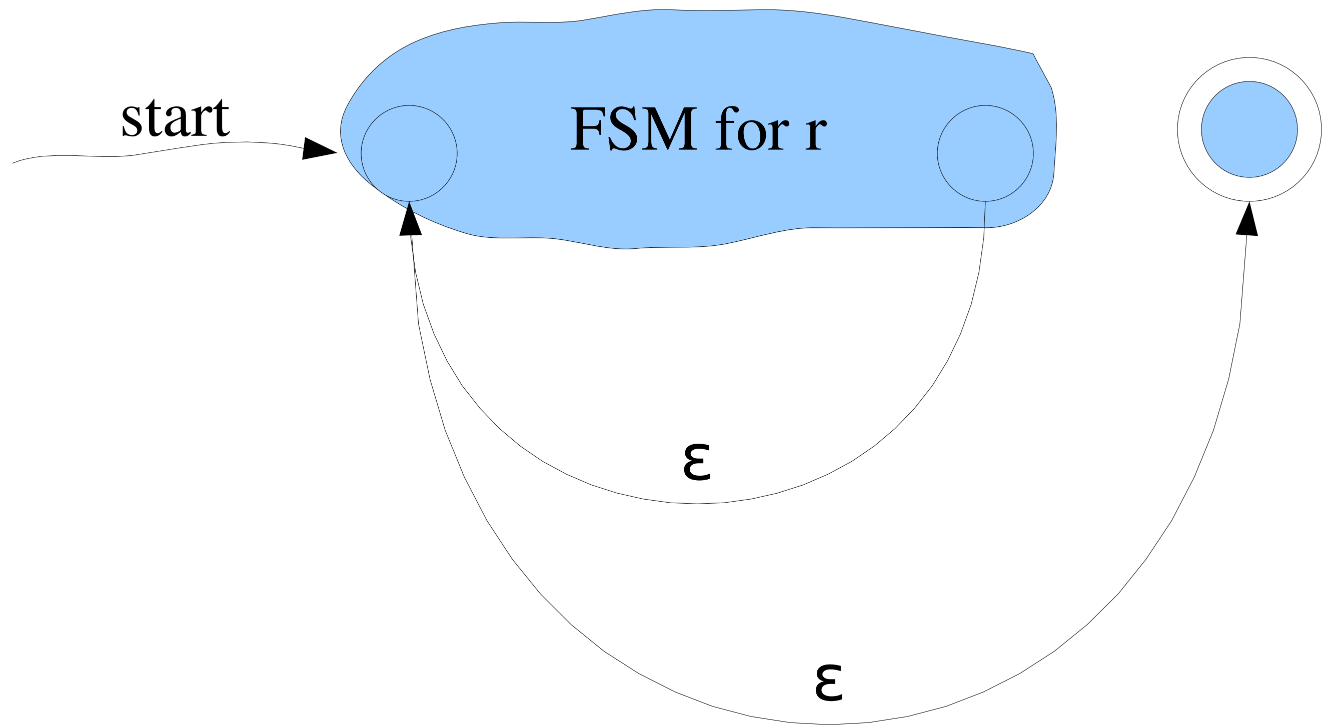
Carleton UNIVERSITY
Canada's Capital University

Concatenation

- rs

Kleene Closure

- r*



NFA to DFA

- So far
  - We can express token patterns as RE
  - We can convert REs to NFA
- NFAs are hard to use
  - Given an NFA F and a string s, it is difficult to test if F accepts s
- Instead, we first convert the NFA into a *deterministic finite automaton*
  - No ε transitions
  - No repeated labels on outgoing edges

Carleton UNIVERSITY
Canada's Capital University

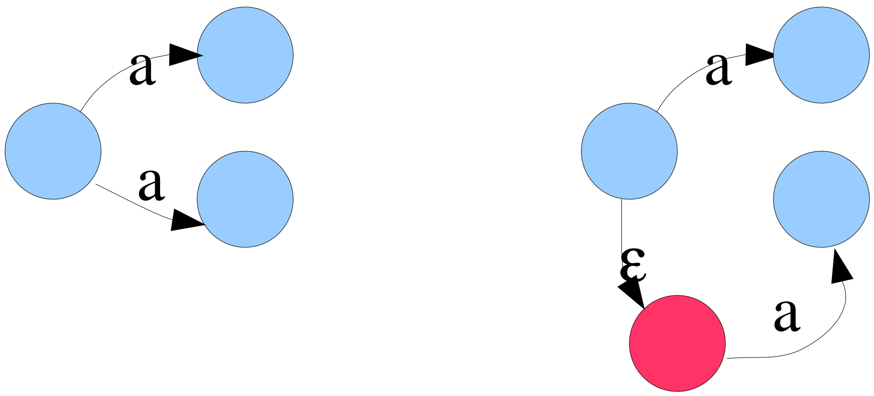
Converting an NFA into a DFA

- Converting an NFA into a DFA is easy but sometimes expensive
- Suppose the NFA has n states $1,\ldots,n$
- Each state of the DFA is labelled with one of the 2^n subsets of $\{1,\ldots,n\}$
- The DFA will be in a state whose label contains i if the NFA could be in state i
- Any DFA state that contains an accepting state of the NFA is also an accepting state

Carleton UNIVERSITY
Canada's Capital University

NFA 2 DFA – Sketch of Algorithm

- Step 1 - Remove duplicate edge labels by using ε transitions



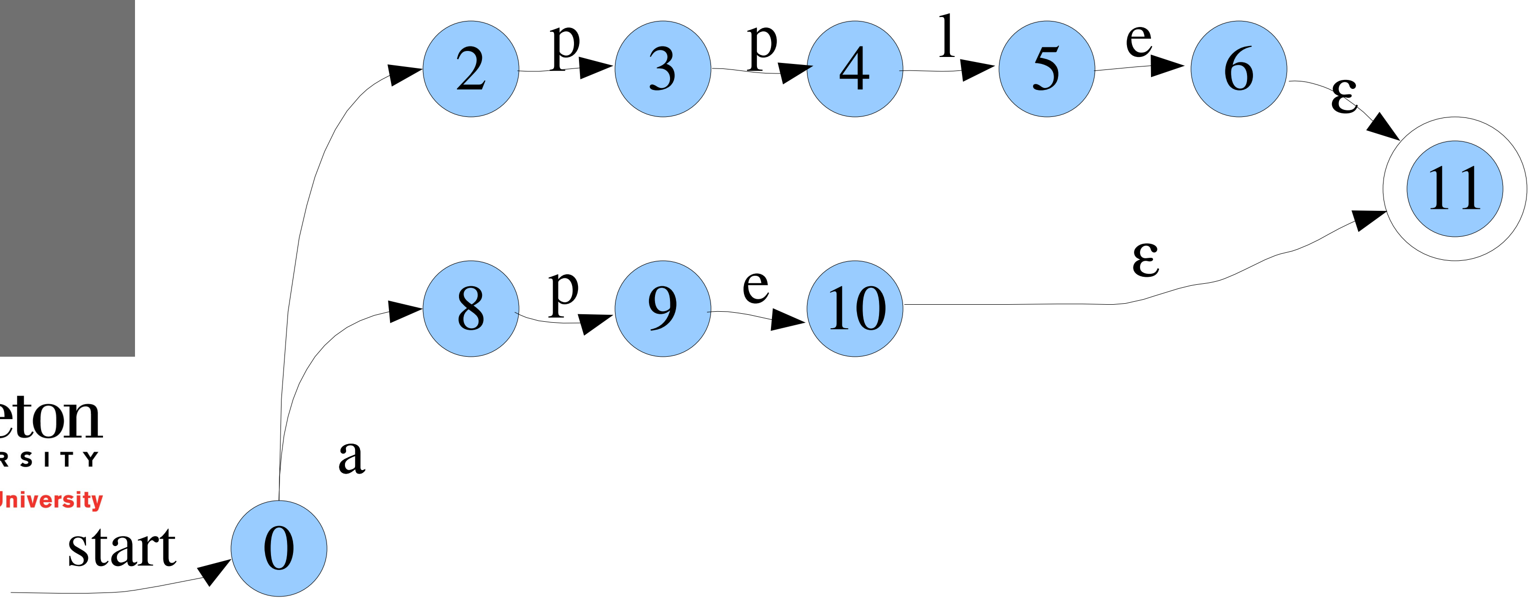
NFA 2 DFA

- Step 2: Starting at state 0, start expanding states
  - State *i* expands into every state reachable from i using only ε-transitions
  - Create new states, as necessary for the neighbours of already-expanded states
  - Use a lookup table to make sure that each possible state (subset of {1,...,n}) is created only once

Carleton UNIVERSITY
Canada's Capital University

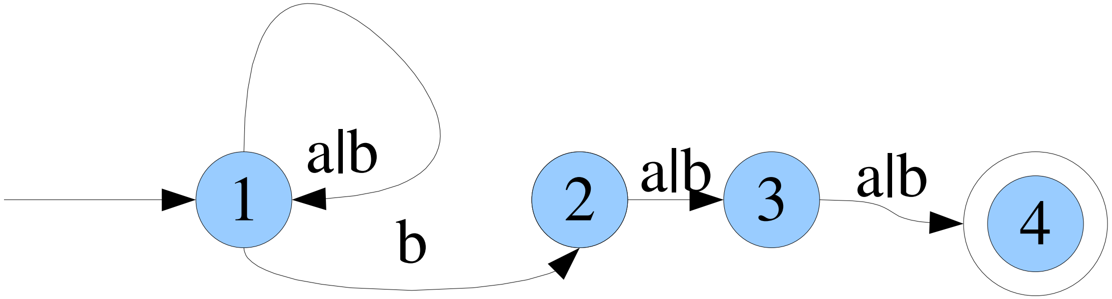
Example

- Convert this NFA into a DFA



Example

- Convert this NFA into a DFA



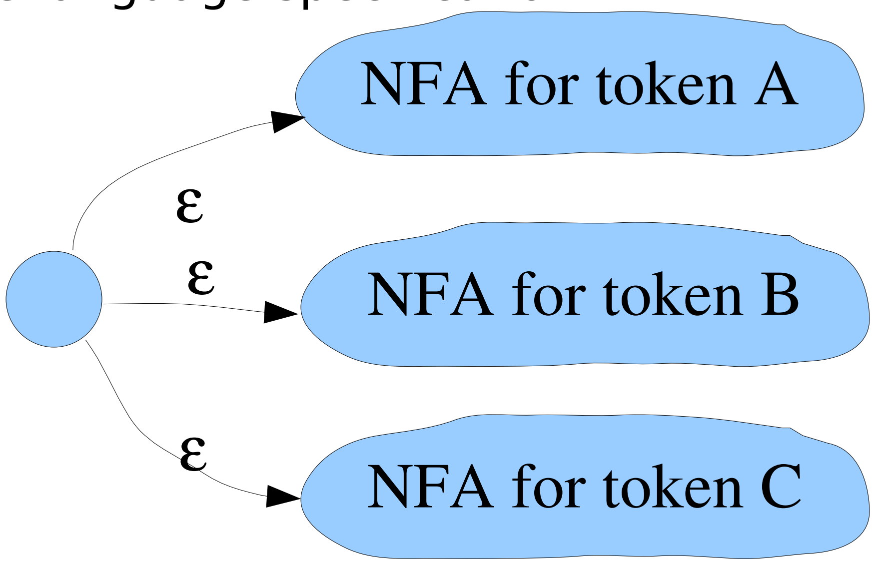
From REs to a Tokenizer

- We can convert from RE to NFA to DFA
- DFAs are easy to implement
  - Using a switch statement or a (hash)table
- For each token type we write a RE
- The lexical analysis generator then creates a NFA (or DFA) for each token type and combines them into one big NFA

Carleton UNIVERSITY
Canada's Capital University

From REs to a Tokenizer

- One giant NFA captures all token types
- Convert this to a DFA
  - If any state of the DFA contains an accepting state for more than 1 token then something is wrong with the language specification



Summary

- The Tokenizer converts the input character stream into a token stream
- Tokens can be specified using REs
- A software tool can be used to convert the list of REs into a tokenizer
  - Convert each RE to an NFA
  - Combine all NFAs into one big NFA
  - Convert this NFA into a DFA and the code that implements this DFA

Carleton UNIVERSITY
Canada's Capital University

Other Notes

- REs, NFAs, and DFAs are equivalent in terms of the languages they can define
- Converting from NFA to DFA can be expensive
  - An n-state NFA can result in a 2^n state DFA
- None of these are powerful enough to parse programming languages but are usually good enough for tokens
  - Example: the language { $a^n b^n$: n = 1,2,3,...} is not recognizable by a DFA (why?)

Carleton UNIVERSITY
Canada's Capital University