COMP4804 Important Facts

Union of Events and Boole's Inequality. For any events A and B

$$\begin{aligned}\Pr\{A \text{ or } B\} &= \Pr\{A\} + \Pr\{B\} - \Pr\{A \text{ and } B\} \\ &\le \Pr\{A\} + \Pr\{B\} \ .\end{aligned}$$

Conditional Probability.

$$\Pr\{A \mid B\} = \frac{\Pr\{A \text{ and } B\}}{\Pr\{B\}}$$

Another useful way of writing this is

$$\Pr\{A \text{ and } B\} = \Pr\{A \mid B\} \Pr\{B\} \ . \tag{1}$$

Independence. We say that A and B are *independent* if and only if

$$\Pr\{A \mid B\} = \Pr\{A\}$$

If A and B are independent then (1) becomes

$$\Pr\{A \text{ and } B\} = \Pr\{A\} \Pr\{B\} \quad \text{(Only if } A \text{ and } B \text{ are independent!)}$$

Expected Value. For a random variable X

$$\mathrm{E}[X] = \sum_x x \Pr\{X = x\} \ .$$

Linearity of Expectation. For any random variables X and Y

$$\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y] \ .$$

More generally, for any random variables $X_1, \ldots, X_n$

$$\mathrm{E}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} \mathrm{E}[X_i] \ .$$

Linearity of expectation, in combination with *indicator variables*, is extremely useful for things we can count.

Markov's Inequality. For any *non-negative* random variable X,

$$\Pr\{X > t\mathrm{E}[X]\} \le 1/t \ .$$

Bernoulli and Binomial Random Variables. A Bernoulli(p) random variable is a random variable that is equal to 1 with probability p and 0 with probability $1-p$. If X is a Bernoulli(p) random variable then $E[X] = p$. A binomial(p,n) random variable is the sum of n *independent* Bernoulli(p) random variables. If B is a Bernoulli(p,n) random variable then $E[B] = pn$. Also, don't forget Chernoff's bounds:

$$\Pr\{B \ge (1+\epsilon)np\} \le e^{-\epsilon^2 np/3}$$

and

$$\Pr\{B \le (1-\epsilon)np\} \le e^{-\epsilon^2 np/2}$$

Beware: Chernoff's bounds are only for binomial random variables. In particular you must make sure that the X_is are independent!