Entropy

Let $Pr : [n] \to [0,1)$ be a probability function.

Let $p_i = Pr(i)$ for each $i \in [n]$.

Then $H = \sum_{i=1}^{n} p_i \log \frac{1}{p_i}$ is the (Shannon) entropy of $p_1 \dots p_n$.

H measures uncertainty about the outcome.

↘ information learned when the outcome is revealed.

Maximum value of H is $\log n$, when $p_1 = p_2 = \dots = p_n = \frac{1}{n}$.

$$H = \sum_{i=1}^{n} \frac{1}{n} \cdot \log \frac{1}{1/n} = \sum_{i=1}^{n} \frac{1}{n} \log n = \log n.$$

H can be arbitrarily close to zero.

$p_1 = 1 - \varepsilon(n-1)$, $p_2 = p_3 = \dots = p_n = \varepsilon$.

$$H = \underbrace{\underbrace{(1-\varepsilon(n-1))}_{\to 1} \cdot \underbrace{\log \frac{1}{1-\varepsilon(n-1)}}_{\to 0.}}_{\to 0.} + \underbrace{(n-1)\varepsilon \cdot \underbrace{\log \frac{1}{\varepsilon}}}_{\to 0}$$