length of codeword for x. $H(R) = \mathbb{E}(\text{length of codeword})$

· **Entropy:** $H(R) = \sum_{x \in R} \Pr(x) \cdot \underbrace{\log \frac{1}{\Pr(x)}}_{\text{length of codeword for } x}$.

· Measures information or uncertainty about the distribution.

· **Markov chains**

- Transition matrix $P = (p(i,j))_{i,j \in [n]}$.

probability of moving to state j given that you are currently in state i

$$\sum_{j=1}^{n} p(i,j) = 1 \quad \text{for each } i \in [n].$$

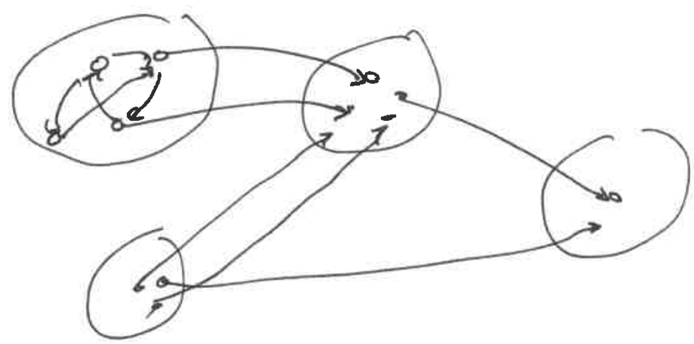
row sums add to 1.

row vector c gives stationary probabilities if

$$c \cdot P = c$$

~~Edges~~

- If we treat each strongly-connected component as a single node, then the edges between components define a directed acyclic (multi-) graph.



This graph has one or more sinks. (no outgoing edges).

Non-sink components are <u>transient</u> (stationary probability 0)

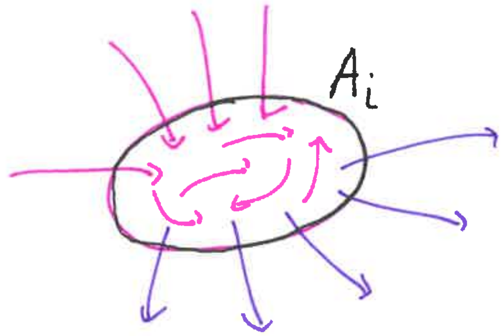
Hack: Introduce a teleport probability. t.

$$p(i,j) = \begin{cases} \frac{t}{n} & \text{if } \vec{ij} \notin E(D). \\ \frac{t}{n} + \frac{1-t}{\deg^+(i)} & \text{if } \vec{ij} \in E(D) \end{cases}$$

With this hack, there is always a unique globally stable stationary distribution.

$$q_{i,out} = \sum_{v \in A_i} \sum_{w \notin A_i} p(v,w) \cdot c(v).$$

$$q = \sum_{i=1}^{\ell} q_{i,out}.$$



$$P_i = q_{i,out} + \sum_{v \in A_i} c(v)$$

$$Q = \left\{ \frac{q_{1,out}}{q}, \frac{q_{2,out}}{q}, \ldots, \frac{q_{\ell,out}}{q} \right\}.$$

$$P_i = \left\{ \frac{q_{i,out}}{P_i}, \frac{c(v_{i,1})}{P_i}, \frac{c(v_{i,2})}{P_i}, \ldots, \frac{c(v_{i,|A_i|})}{P_i} \right\}.$$

where $\{v_{i,1}, v_{i,2}, \ldots, v_{i,|A_i|}\} = A_i$.

- InfoMap Algorithm tries to find $A_1 \ldots A_\ell$ that minimizes $L(\mathcal{A})$.

- Uses rounds of local improvements interleaved with compression, just like the Louvain Algorithm.

- Unlike Louvain (and other graph-modularity-based algorithms), $L(\mathcal{A})$ doesn't have a resolution problem.